

INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT First Year
2015-16 Semester II

Nonparametric Inference and Sequential Analysis:
Midterm Examination

Total Marks 30.

Duration: 3 hours

1. Let (X_1, \dots, X_n) be a random sample from the uniform distribution on the interval $[0, 1]$ and let $R = X_{(n)} - X_{(1)}$, where $X_{(i)}, i = 1, 2, \dots, n$ is the i th order statistic. Find the limiting distribution of $2n(1 - R)$. 5
2. Define influence function of a statistical functional. Find the influence function of a quantile of a distribution. 2+5
3. Let X_1 and X_2 be independent and identical copies of X which have an absolutely continuous distribution function F . Define parameter defined by

$$\Delta = E|X_1 - X_2|.$$

Find a U-statistics based estimator of Δ , and find the variance of the estimator when $F(x) = 1 - e^{-x}$. 3+5

4. Define consistency of a test. If F is a continuous distribution function with unique median θ , then show that the sign test is consistent for tests on θ . 5
5. Express Wilcoxon-Mann-Whitney test as U-statistics. 5
6. Find the distribution of Run test under the null hypothesis of equality of two distribution. 5
7. Give an outline of the proof of finding the asymptotic distribution of Kolmogorov-Smirnov one sample test. 5